Modelling foveated vision in Matlab image processing

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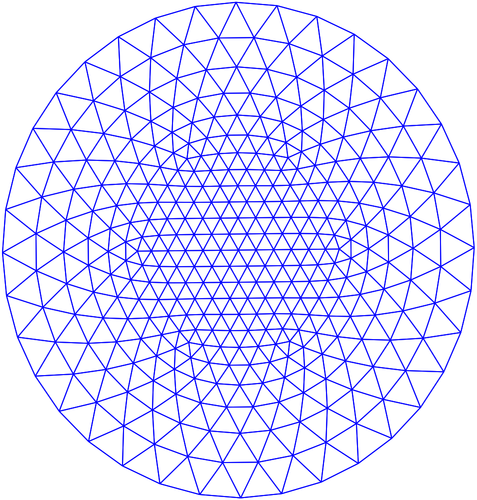
Fovea example

hexagonal side length 6,
6 circles in the periphery.
The numbers used in the examples are:

- side length 20
- peripheral circles 64
- 8941 positions for a vision field of 10°
- 512×512 images.



Essential questions

Tasks which are very difficult on foveate grids

- What is a line?

- What is a straight line?
- What is a circle?
- What is a right angle?
- What is a parallel?
- How are lengths compared?

- How is motion handled?

Positions in eye space and perception space

- eye (luminance) space: real values ($-1/2 \leq x \leq 1/2$) on the foveate grid
- perception space: similar, but much more positions
- values of luminance, colour, distance and velocity

Complex numbers denote positions in 2dimensional space.
Rules (i denotes the imaginary unit):

$$|a + \mathrm{i} \cdot b| = \sqrt{a^2 + b^2}$$

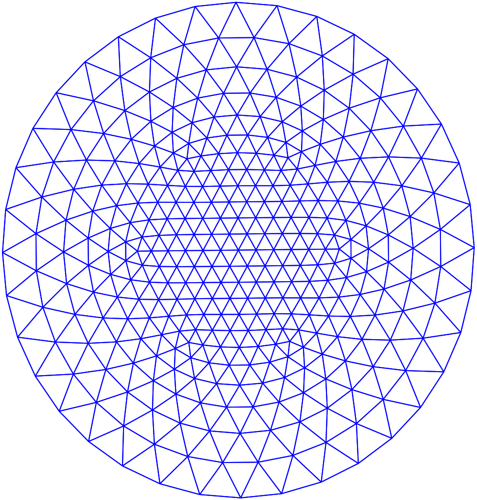
$$\mathrm{Re}\ z = |z| \cdot \cos\ \phi\ ,\ \ \mathrm{Im}\ z = |z| \cdot \sin\ \phi$$

$$\mathrm{e}^{\mathrm{i} \cdot \phi} = \cos\ \phi + \mathrm{i}\ \ \cdot \sin\ \phi$$

ϕ is called the "phase" of z, and $\mathrm{e}^{\mathrm{i} \cdot \phi}$ its "phase factor.
Multiplication by a complex number is rotation by phase, and scaling by magnitude.

Foveate graph

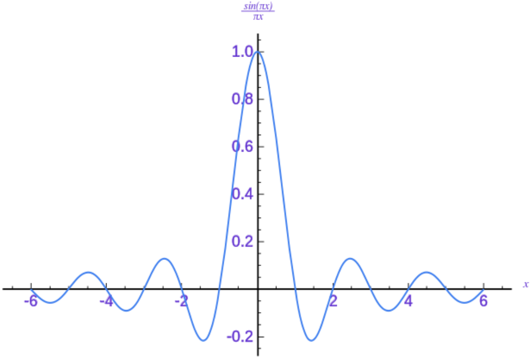
```
[Fp,H] =
hexnet(6,6);
gplot(H,pairs(Fp));
```



Nyquist-Shannon sampling theorem

for band-limited interpolation

- data y_n sampled at $x_n = n \in \mathbb{Z}$
- $f(t) = \sum y_n \cdot \mathrm{sinc}(\pi \cdot (t - n))$
- cardinal sinus function $\mathrm{sinc}(x) = \sin(x)/x$
- 2-dim analog: $\mathrm{jinc}(r) = J_1(r)/r$, in Mathlab $J_1(r) =$ `besselj(1,r)`



Foveate example

discrete Laplace operator

$$\Delta = -(\partial^2/\partial x^2 + \partial^2/\partial y^2)$$

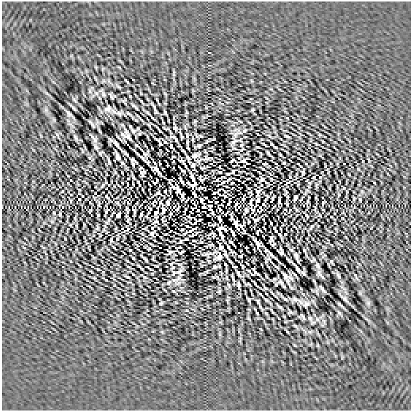
- is zero when a function is harmonic (the mean of adjacent values)
- is nonzero where interesting features are

- Δ_{nm} is the difference of unit matrix and a multiple of adjacency matrix
- $\sum \Delta_{nm} = 0$, for each column m
- $\sum z_n \cdot Delta_{nm} = 0$
- $\sum |z_n|^2 \cdot Delta_{nm} = 1$
- $\sum ((\mathrm{Re} z_n)^2 - (\mathrm{Im} z_n)^2) \cdot Delta_{nm} = 0$

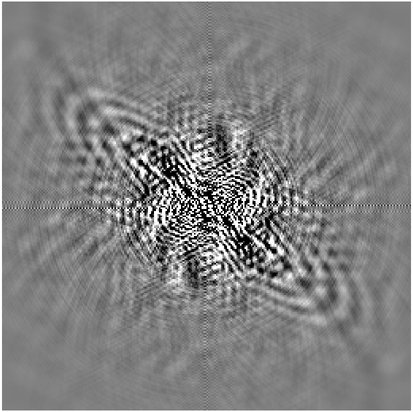
Fourier transformation on a foveate grid

interpolate to Cartesian – `fft2` – operate on FT – `fft2` – foveate

real part of original FT

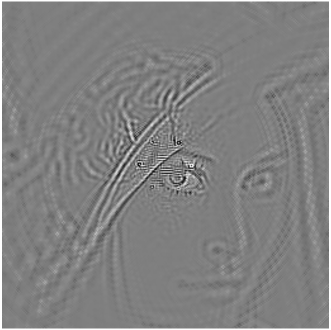


foveate FT



Saccadic integration

- predict translation in perception space
- if motion present, prediction of motion also
- foveate prediction
- difference between prediction and eye data from new fixation
- interpolate difference, add to prediction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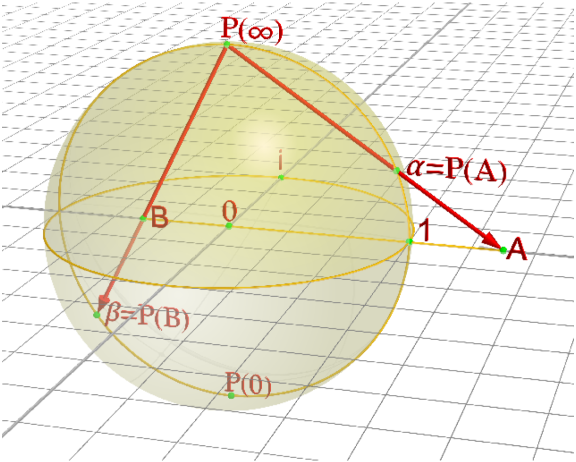
Stereographic projection and Riemann sphere

- complex plane extended by ∞
- 1-dimensional complex projective space
- Möbius transformations

$$z \mapsto \frac{\alpha \cdot z + \beta}{\gamma \cdot z + \delta}$$

- related to 3d Euclidean group
- Fourier analysis (J. Turski)



Schrödinger Equation of the Harmonic Oscillator

$$-\mathrm{i} \;\cdot\; \mathrm{d}/\mathrm{d}t\ \Psi = \Delta\Psi + |x|^2 \cdot \Psi$$

- describes the wave function of a harmonic oscillator in wave mechanics
- $\Psi(x, t + \pi/2)$ is the Fourier transform of $\Psi(x, t)$
- $\Psi(x, t + \pi) = \Psi(-x, t)$ (space reversal)
- the equation involves only local operators
- solution by discretisation possible
- within reasonable restrictions for a physiological system
- iterative refinement by negative feedback from the reverse

Conclusions

- Fourier transform needed in foveate vision.
- Fourier transform is feasible under reasonable restrictions.
- There are signs, that it is happening. (Hermann grid)
- (Eye) movements are the link between the geometry of the perception space and the mathematical groups of the physical world.

More on this on
`http://pkeus.de/dokuwiki/doku.php?id=en:foveate`
or simply `http://pkeus.de/dokuwiki`